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Acoustical Properties of Trumpets

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Acoustical properties of the B♭ trumpet were investigated. By comparing two trumpets judged to have good and poor musical quality, respectively, it was found that the air leakage at the pistons reduced the Q value of the resonance. An irregular frequency distribution of the higher normal modes of the trumpet was also studied. The effect of the connecting condition of the tube and horn of the trumpet on the harmonic relation of the normal modes of the air column was studied. The measured resonance frequencies were found to agree well with the maximum driving point radiation resistance computed for various combinations of tube and horn.

INTRODUCTION

The player of a wind instrument generally thinks of a "good instrument" as one that can be blown easily and has a fine tone quality. What in physical terms is a good instrument? Among recent researches on the physical characteristics of brass wind instruments may be listed the investigation of the force the player exerts against the mouthpiece of a trumpet[1] and that of the resonance characteristics of the cornet.[2]

In the present work the frequency response of several B♭ trumpets was measured, and it was found that the resonance characteristics are a measure of the quality of the trumpet. The air leakage from the piston part of the trumpet plays an important role.

It was found that the frequencies of the normal modes of the trumpet are not integral multiples of the fundamental frequency-a fact also noted by Webster.[3] This phenomenon must depend upon the shape of the trumpet. In the present work the effects of the shape of horn were studied when it was connected to a straight tube; the resonance frequencies for special cases such as conical, exponential, and catenoidal horns were calculated. The measured resonance frequencies were found to agree well with those calculated by the theory.

MEASUREMENT OF ACOUSTICAL CHARACTERISTICS

The acoustical properties of two trumpets were studied by several methods. The one denoted as trumpet *A* was judged by a musician to be of poor quality, and the other, trumpet *B*, was judged to be of good quality. The wave forms of the sounds as played by a musician were photographed by the help of an oscillograph, and the sound spectra were analyzed by a heterodyne type

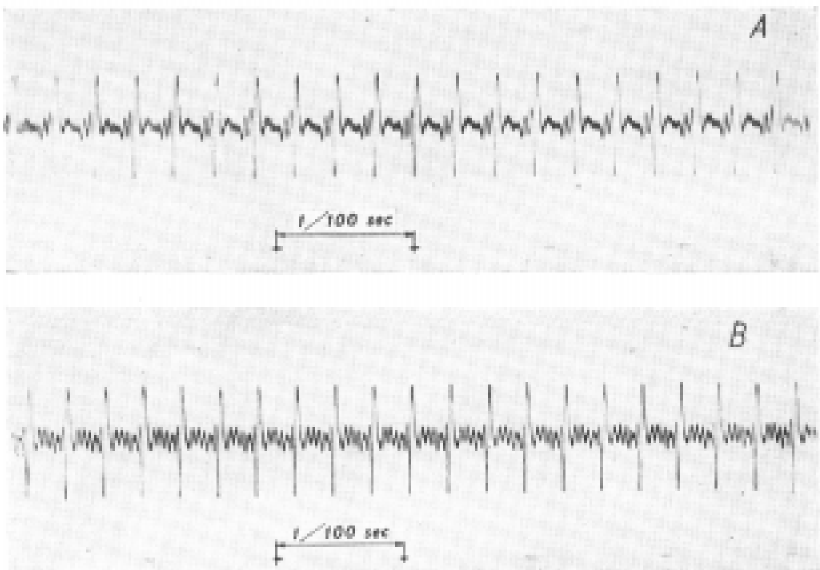

FIG.1. Wave forms of note f' (fundamental frequency roughly 340cps) photographed by an oscillograph recorder. Trumpet A was of poor quality, trumpet B of good quality.

[1] H.W.Henderson, J.Acoust.Soc.Am. 14, 58-64(1942).
[2] J.G.Woodward, J.Acoust.Soc.Am. 13, 156-159(1941).
[3] J.C.Webster, J.Acoust,Soc.Am. 21, 208(1949).

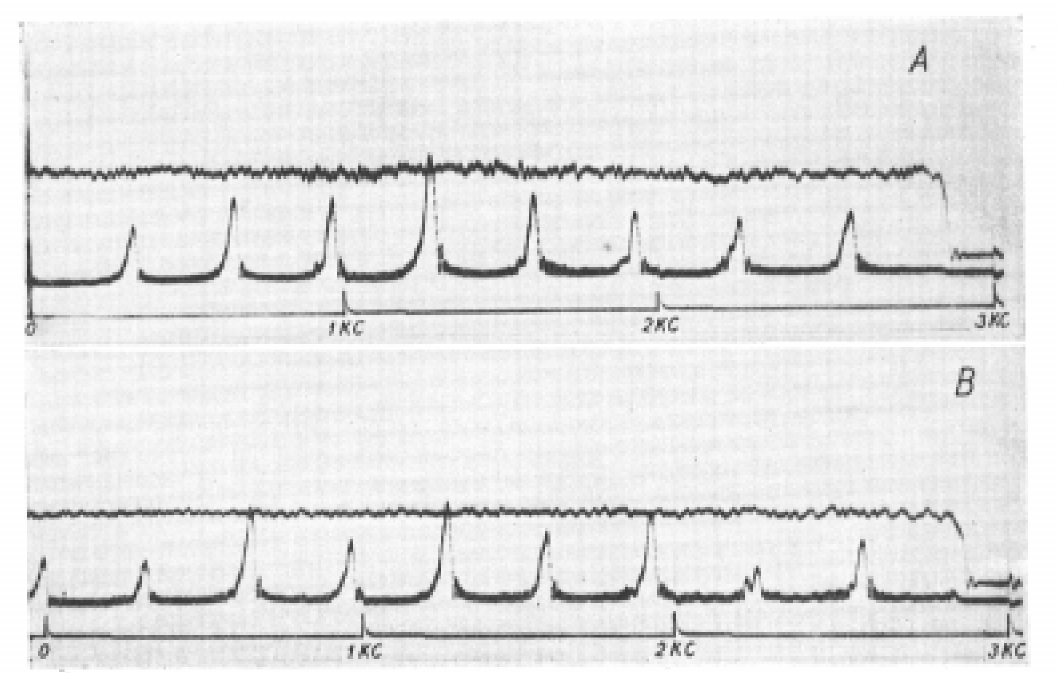

FIG.2. Sound spectra of note f' (fundamental frequency roughly 340 cps), played on trumpets A and B. The peak at zero frequency in spectrum B should be disregarded. Lines above spectra indicate the total amplitude of the sound being played.

frequency analyzer for the successive notes c', d', e', f', etc.(notes written at the bottom of the treble clef). The two trumpets were played by a skillful musician in an anechoic chamber.

Figure 1 shows the wave forms for the note f' as produced by the two instruments, and Fig.2 shows the corresponding spectra. Although the wave forms and spectra differ for the two instruments, it could not be concluded from these data alone that trumpet B was better than trumpet A . We noticed, moreover, that both the sound amplitude and the wave form fluctuated during the measurements, even though the player made an effort to blow constantly. The fluctuation of wave form might be caused by small changes in embouchure, so in order to study the normal modes of the trumpet it was necessary to report to a mechanical or electrical device to maintain the instrument in vibration.

The first source of sound tried was a copper reed, placed near to the mouthpiece of the trumpet, which was vibrated by compressed air. The acoustical impedance of the source was evidently affected by that of the trumpet, and the measured frequency response of the trumpet turned out to be exceedingly irregular. It was therefore necessary to adopt a high impedance source.

The arrangement next used to blow the trumpet and to make the measurements is that shown in Fig.3. The high impedance source consisted of a tube filled with copper wires in the manner described by Hunt.[4] It was attached to a Western Electric Type 555 driver excited by a beat-frequency oscillator. The response-frequency characteristic of this source in a free field was constant within ± 1 db in the range from 0.2 kc to 1.3 kc. The tube was attached to the trumpet mouthpiece in the manner indicated at the bottom of Fig.3. The adaptor projected into the mouthpiece a distance of 2mm and was sealed around the outside by modeling clay. The sound thus passed through a small hole whose inside diameter was 2 mm. At the resonance frequencies of the trumpet the impedance of a source is high compared with the load impedance, so that the reaction on the source is thought to be small.

The whole assembly was installed in an anechoic chamber, the bell of the trumpet being at a distance of 50 cm from the velocity microphone (manufactured by the Matsuda Company) which was connected to a sound level recorder. Figure 4 shows the frequency responses of the two trumpets thus plotted automatically as the frequency was increased steadily. The trumpet valves. Also shown in the figure is the response curve for another trumpet A' , which is almost identical to A except that its valves were very carefully finished.

The frequency at resonance and the values of Q determined from the frequency response curves are shown in Table I. The values of Q were computed in the usual way by taking the ratio of the center frequency to the difference in the two adjacent frequencies at which the response was down 3 db. Frequencies of resonance were checked by comparison with variable tuning forks made by the Max Cohl Company.

[4] F.V.Hunt, J.Acoust.Am. **10**, 216-227 (1939).

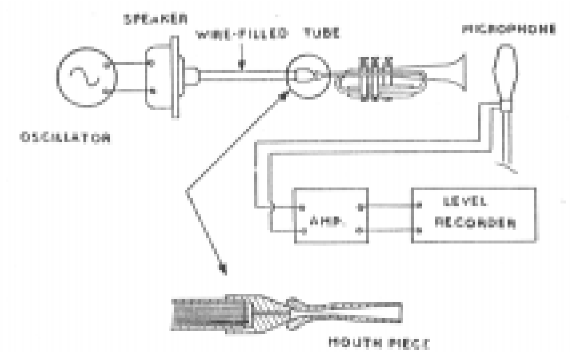

FIG.3. Schematic diagram of the arrangement for the measurement of the frequency response of a trumpet.

The resonant frequencies given in the table are about 5 percent higher than the corresponding frequencies when the trumpets were played in the normal manner. Evidently there is some difference between the acoustical impedance of the artificial source as it is connected to the mouthpiece and that which exists with the player's mouth. Nevertheless, from the resonance characteristics measured electrically in this way, it is believed that the acoustical properties of the trumpet can be deduced qualitatively. When a sharp resonance exists, presumably the note is stable and can be played easily.

The small values of Q for trumpet A were thought to be due to leakage of air around the pistons. The bell of each trumpet was accordingly closed with a rubber plug, and the mouthpiece was connected to a compressed air reservoir so that the air leakage could be measured. Figure 5 shows what had been anticipated from the frequency response curves: namely, that there was much more leakage of air from trumpet A than for trumpet B.

Form Table I it was noticed that the Q values of the odd numbered modes of vibration for trumpet A were especially small. This is apparently related to the fact that positions of maximum sound pressure must exist somewhere in the middle part of the instrument where the leaky valves are located. In view of these results, we measured the leakage and resonance characteristics of trumpet A' which was almost identical with A except that the valves of the former were more carefully finished. Figure 5 shows that the leakage of air was less for trumpet A' than for trumpet A, and Fig.4 shows that the resonances were more pronounced for trumpet A'.

As shown in Table I, the differences in the successive resonance frequencies were considerably irregular; also wide and narrow intervals of A correspond, respectively, to those of B. Since these irregularities are often found in the trumpet, an improvement in this respect is highly desired by players. Accordingly, the effect of the construction of the trumpet upon the frequency response was studied in some simplified cases.

COMBINATIONS OF TUBE AND HORN

Cylindrical tubes of different length were prepared and also a conical horn. The flaring angle of the horn was 40 degrees; the diameters of the mouth and throat of the horn were 15 cm and 2.0 cm, respectively. The frequency responses of combinations of tubes of various length attached to the horn were measured. The method

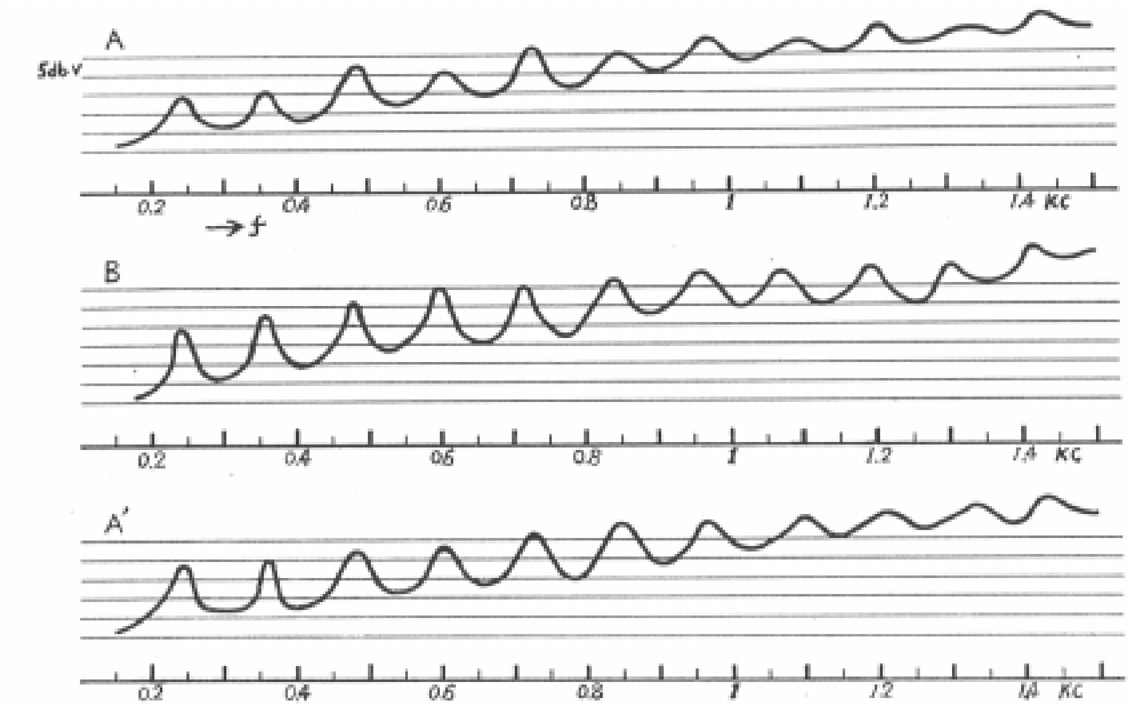

FIG.4. Freqiemcu responses of trumpets A and B for open valves. Tumpet A′ was almost identical to A, except that the valves were carefully finished.

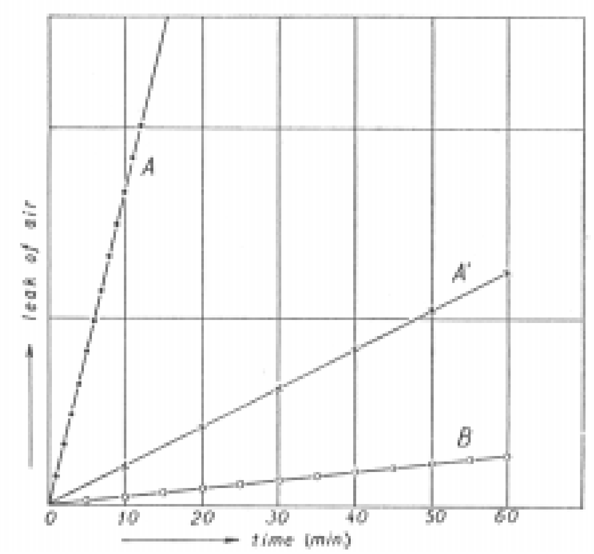

FIG 5. Amount of air leakage from trumpets as a function of time.

of experiment was similar to that given above. It is evident from Fig.6 that the frequency distribution of the normal modes for such tube-cone combinations is exceedingly irregular. The frequency response, thus observed, should correspond to the radiation resistance at the source side of the tube. For simplicity, it was assumed that the impedance at the mouth of the horn (the large end) is given adequately for an opening surrounded by an infinite baffle,[5]

$$z = \rho c\left[1 - 2J_1(w)/w - iM(w)\right] , \qquad (1)$$

TABLE I. Measured resonance frequencies and Q values for "open" notes. Trumpet A was of poor quality, B of good quality.

Trumpet A			Trumpet B		
Resonance frequency cps	Frequency difference cps	Q	Resonance frequency cps	Frequency difference cps	Q
…			…		
				111	
243		5.8	246		13.3
	118			124	
361		9.6	357		15.5
	125			116	
486		16.8	481		19.2
	116			121	
602		8.2	597		28.4
	124			123	
726		23.4	718		28.7
	126			115	
852		12.2	841		25.5
	118			116	
971		21.1	956		27.3
	121			117	
1092		…	1072		20.8
	118			119	
1210		16.3	1189		19.5
…			1308		20.4
…			1419		

[5] P.H.Morse, Vibration and Sound (McGraw-Hill Book Company, Inc., New York, 1948), second edition, p.255.

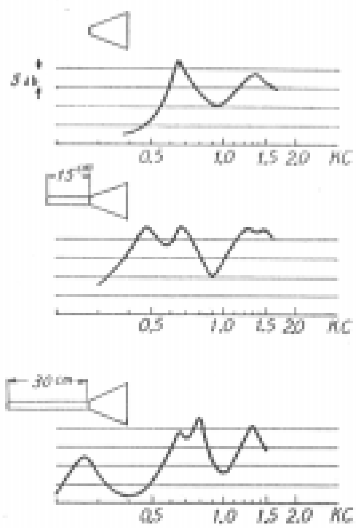

FIG 6. Frequency responses of the combination of conical horn and tubes.

where $w = 2\omega a/c$, a is the diameter of the mouth, ω the angular frequency, c is the velocity of sound, and ρ is the density of air. J_1 and M are Bessel's function of the first order and Struve's function, respectively. In general, the impedance of the horn [6] is represented by

$$z = \rho c\big[\gamma \coth(\psi + i\omega\gamma x/c) + (ic/\omega h)\tanh\{(x/h)+\varepsilon\}\big]^{-1} . \qquad (2)$$

Here $\gamma^2 = 1-(c/\omega h)^2$, ε and h are parameters determining the shape of the horn. For the conical horn $\varepsilon = x_0/h + i\pi/2$ and h tends to infinity. Then the impedance at x is

$$z(x) = \rho c\left[\coth(\psi + i\omega x/c) + ic/\omega(x+x_0)\right]^{-1} , \qquad (3)$$

where x_0 is the distance from the throat of the horn to the apex (Fig.7). It we set $x = l$ in Eq.(3), it is the impedance at the mouth of the horn z_{III} and is equal to Eq.(1). Real and imaginary parts of Eq.(1) for various values of w are known from a table,[7] and it is possible to calculate $\tanh(\psi + i\omega l/c)$ in terms of θ_1 and x_1

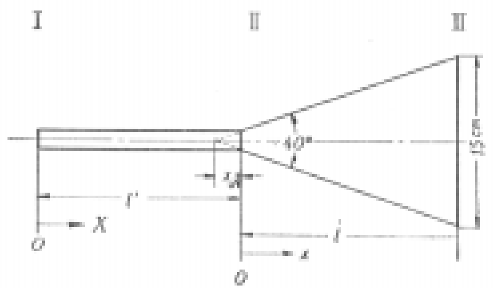

FIG 7. Schematic diagram of the combination of tube and horn, indicating dimensions used in theoretical calculation.

[6] See reference 5, p.284.
[7] G.N.Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, Cambridge), p.666.

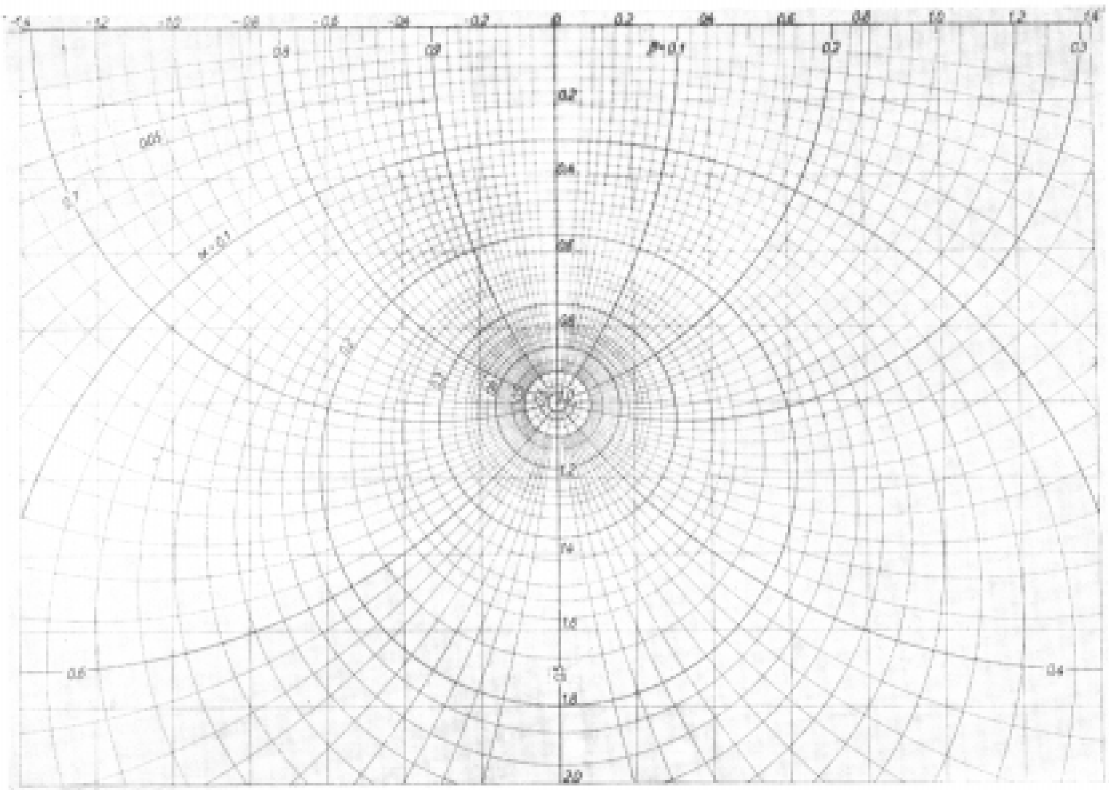

FIG.8. Bipolar plot of α and β against impedance ratio $z/\rho c = \tanh\{\pi(\alpha - i\beta)\} = \theta - i_x$, $0 < \theta < 2.0$, $-1.4 < x < 1.4$.

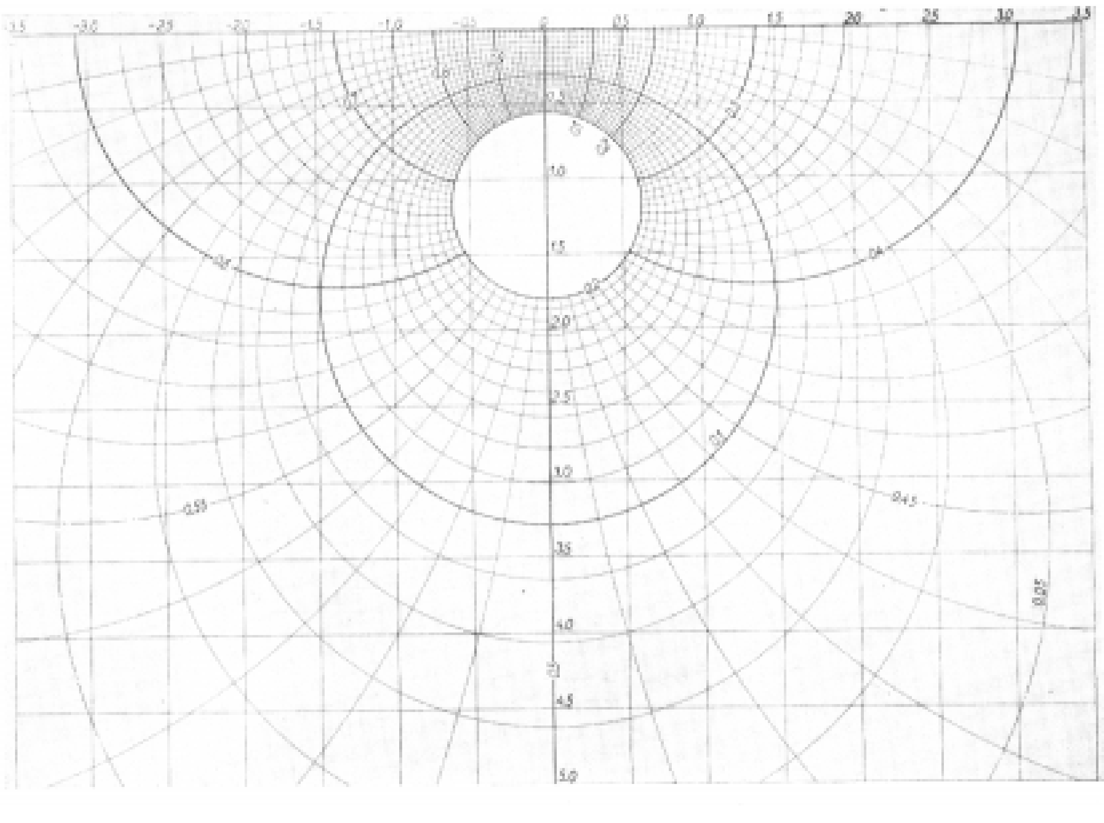

FIG.9. Bipolar plot of α and β against impedance ratio $z/\rho c = \tanh\{\pi(\alpha - i\beta)\} = \theta - ix$, $0 < \theta < 5.0$, $-3.5 < x < 3.5$.

which are given by the following formula:

$$\tanh(\psi + i\omega l / c) = \theta_1 - i_{x1}\,,$$

$$\theta_1 = \frac{1 - 2J_1(w)/w}{\{1 - cM(w)/w(l + x_0)\}^2 + \{c[1 - 2J_1(w)/w]/\omega(l + x_0)\}^2}\,, \qquad (4)$$

$$x_1 = \frac{M(w) - c\{[1 - 2J_1(w)/w]^2 + [M(w)]^2\}/\omega(l + x_0)}{\{1 - cM(w)/\omega(l + x_0)\}^2 + \{c[1 - 2J_1(w)/w]/\omega(l + x_0)\}^2}\,.$$

By the chart of the hyperbolic tangent of a complex quantity [8] (Figs.8 and 9), α_1 and β_1 are determined from the set of values θ_1, x_1, where

$$\tanh\{\pi(\alpha_1 - i\beta_1)\} = \theta_1 - ix_1\,.$$

If we set $\beta_1' = \beta_1 + (\omega l / \pi c)$, then $\tanh\psi$ can be calculated by

$$\tanh\psi = \tanh\{\pi(\alpha_1 - i\beta_1')\} = \theta_2 - ix_2\,.$$

Then the impedance at the throat of the horn,

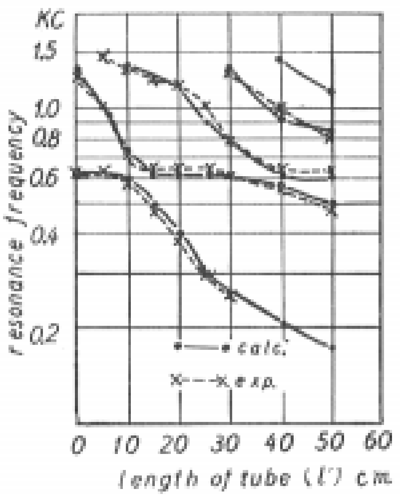

FIG 10. Resonance frequencies of radiation resistance of the combination of a conical horn and tubes of various lenghts.

obtained from Eq.(3) by putting $x = 0$,

$$z_{II}/\rho c = \tanh\{\pi(\alpha_2 - i\beta_2)\} = \theta_3 - ix_3\,,$$

$$\theta_3 = \theta_2 / [(1 + cx_2/\omega x_0)^2 + (c\theta_2/\omega x_0)^2]$$

$$x_3 = [x_2 + c(\theta_2^2 + x_2^2)/\omega x_0] / [(1 + cx_2/\omega x_0)^2 + (c\theta_2/\omega x_0)^2]\,. \qquad (5)$$

The acoustic impedance of the tube at the distance X from the end is

$$z(X) = \rho c \tanh(\psi + i\omega X / c)\,. \qquad (6)$$

If we set $X = l'$ in Eq.(6), it is the impedance at the other end of the tube z_{II} and is equal to Eq.(5). Similarly as before, from the values of α_2 and β_2' where $\beta_2' = \beta_2 + (\omega l' / \pi c)$, we can calculate the impedance at the source side of the tube z_I

$$z_I / \rho c = \tanh\psi = \tanh\{\pi(\alpha_2 - i\beta_2')\} = \theta_0 - ix_0\,. \qquad (7)$$

[8] See reference 5, p.453. Hyperbolic tangents of complex quantities were calculated in detail, which are shown in Figs.8 and 9.

The frequencies for the maximum radiation resistance at the source are plotted in Fig.10 for different lengths of cylindrical tubes. The dotted curves represent experimentally determined frequencies of resonance. They are in good agreement with each other.

From Fig.10, the irregular frequency distribution of the higher modes is apparent. This irregularity seems to be related to the discontinuity in shape where cylindrical tube and conical horn are connected. If we choose the exponential or catenoidal horn, the discontinuity would be less than in the case of the conical horn. Similar calculations were accordingly performed for such horns. The impedance of the exponential horn at the distance x from the throat is

$$z(x) = \rho c\,[\gamma \coth(\psi + i\omega\gamma x / c) + ic/\omega h]^{-1}\,, \qquad (8)$$

and that of the catenoidal horn is

$$z(x) = \rho c[\gamma \coth(\psi + i\omega\gamma x / c) + (ic/\omega h)\tanh(x/h)]^{-1}\,. \qquad (9)$$

The values of h's in these formulas are chosen as 7.8cm and 6.0cm, respectively, and the diameters of the throat and the length of each horn are the same as those of conical horn. The frequencies of maximum radiation resistance at the source are plotted in Figs.11 and 12. The dotted curves in Fig.12 represent experimental results. In Fig.12 the resonance frequencies at various tube lengths are distributed regularly except for the fundamental frequency. In connecting the tube and

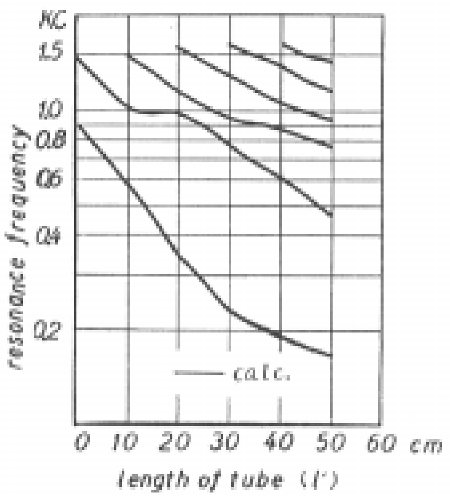

FIG 11. Resonance frequencies for the combination of a exponential horn and tubes of various lengths.

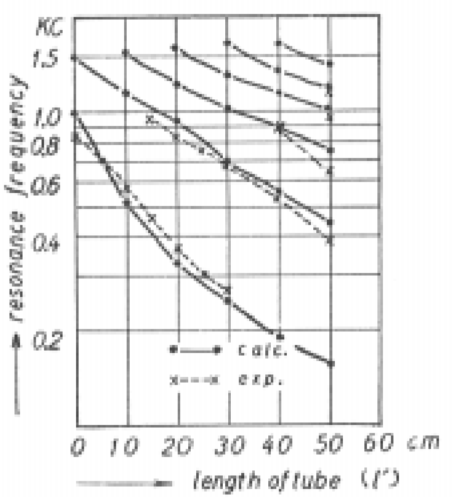

FIG 12. Resonance frequencies for the combination of an catenoidal horn and tubes of various lenghts.

the horn as in a wind instrument, the discontinuity at the connecting part must be avoided; the catenoidal horn seems to be best from this point of view.

SUMMARY

It is very convenient to use the electrical method to study the resonance characteristics of the trumpet qualitatively.

The electrical blowing method does not give the same results as normal playing; the resonance frequencies measured in this way are somewhat higher.

The air leakage at the pistons reduces the Q-value of the resonance: it must be made as small as possible. The distribution of the resonance frequencies of the trumpet is fairly irregular; this phenomenon is partly due to the discontinuity at the tube-horn connection. The sound wave reflected at this part produces inhormonic modes in the trumpet.

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